Calc AB Ch 2 Test Practice

Name: Oct 5, 2022 Block: Seat:

Chapter 2 skills Check List:

1 ...... Average Rate of Change versus Instantaneous Rate of Change (secant line slope vs tangent line slope)

2 ...... Conditions for Continuity:

- i. $f(c)$ exists
- ii. $\lim_{x\to c} f(x)$exits (one sided limits must agree)
- iii. $\lim_{x\to c} f(x) = f(c)$

3 ...... Conditions for Differentiability:

- i. Continuous at $f(c)$
- ii. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exits (one sided limits must agree)
- iii. not vertical (no slope)

4 ...... Both limit definitions of Derivative (p 103 and 105)

5 ...... Differentiable at a point or on an open interval (p. 103)

6 ...... Differentiability Implies Continuity (p 106)

7 ...... The Constant Rule (p 110)

8 ...... The Power Rule (p 111)

9 ...... The Constant Multiple Rule (p 113)

10 ...... Sum and Difference Rules (p 114)

11 ...... Derivatives of Sine and Cosine (p 115)

12 ...... Derivative of e^x is e^x

13 ...... Derivative of $\ln x$ is $\frac{1}{x}$

14 ...... Product & Quotient Rule (p 122, 124)

15 ...... Derivatives of tan, cot, sec, csc (p 126- if not by heart, by using sin and cos with quotient rule)

16 ...... Higher Order Derivatives (p 128) like velocity and acceleration

17 ...... Chain Rule (p 134)

18 ...... General Power Rule (p 134)

19 ...... Summary of Differentiation Rules (p 139)

20 ...... Guidelines for Implicit Differentiation (p 145)

21 ...... Related Rates Word Problems (p152).

Delta Math Check List:

1 ...... Delta Math Lab: Delta Math Lab 1-7: Limit Definition of Derivative (4 skills)

2 ...... Delta Math Lab 1-8: Basic Derivative Questions (4 skills)

3 ...... Delta Math Lab 1-9: Power, Product, and Quotient Rules (6 skills)

4 ...... Delta Math Lab 1-10: Basic Chain Rule (5 skills)

5 ...... Delta Math Lab 1-11: Implicit Differentiation (6 skills)

6 ...... Delta Math Lab 1-12: Related Rates (4 skills)

Khan Academy Check List:

1 ...... AP Calculus AB Unit: **Differentiation: definition and basic derivative rules** (Start the unit test and collect up to 2,300 possible Mastery points each time it is 23 questions and should take 46 minute or less)

2 ...... In the AP Calculus AB Unit: **Differentiation: composite, implicit, and inverse functions** are some chapter 2 topics:

- i. Chain Rule Intro (collect 80-100 Mastery Points)
- ii. Chain rule with Tables (collect 80-100 Mastery Points)
- iii. Implicit Differentiation (collect 80-100 Mastery Points)
- iv. Contextual applications of Derivatives Unit topic: Solving Related Related Rates Problems (AP Unit 4.4)

1. Definition of Derivative (2.1)

(a) This table gives select values of the differentiable function f.

x	4	5	6	7
$f(x)$	1	18	35	53

What is the best estimate for $f'(7)$ we can make based on this table?

A. 9.6
B. 18
C. 53
D. 11

Best estimate would be the slope of the secant line... The average rate of change from 6 to 7 is

$$\frac{f(7)-f(6)}{7-6} = \frac{53-35}{7-6} = 18$$

B

(b) What is the average rate of change of $g(x) = 7 - 8x$ over the interval $[3, 10]$?

$$\frac{g(10)-g(3)}{10-3} = \frac{(7-80)-(7-24)}{10-3}$$

$$= \frac{-73-(-17)}{7}$$

$$= -8$$

(c) Use the limit Definition of a derivative to show to show the derivative of $f(x) = 3x^2 - 4x$ is $6x - 4$

$$\lim_{h\to 0} \frac{3(x+h)^2 - 4(x+h) - (3x^2 - 4x)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{4x} - 4h - \cancel{3x^2} + \cancel{4x}}{h}$$

$$\lim_{h\to 0} \frac{6xh + 3h^2 - 4h}{h} = \lim_{h\to 0} 6x + 3h - 4$$

$$= 6x - 4$$

(d) Evaluate $\displaystyle\lim_{h\to 0} \frac{(3+h)^{23} - 3^{23}}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

if $f'(3)$ if $f(x) = x^{23}$

Since $f'(x) = 23x^{22}$

this limit is also $= f'(3) = 23 \cdot 3^{22}$

(see page 103)

(e) Evaluate $\displaystyle\lim_{x\to 2} \frac{4x^3 - 32}{x - 2}$ looks like def. of derivative on page 105

$\displaystyle\lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$ if $f(x) = 4x^3$

note $f(2) = 4(8) = 32$

So if this is equal to the derivative...

$f'(x) = 12x^2$

$f'(2) = 48$

$\therefore$ ... the limit must also be 48

(f) Let $g(x) = \ln x$. Which of the following is equal to $g'(5)$?

A. $\displaystyle\lim_{x\to 5} \frac{\ln(5+x) - \ln(5)}{x-5}$ ✗

B. $\displaystyle\lim_{x\to 5} \frac{\ln(x) - 5}{x-5}$ ✗

C. $\displaystyle\lim_{x\to 5} \frac{\ln(x-5)}{x-5}$ ✗

D. $\displaystyle\lim_{x\to 5} \frac{\ln(x) - \ln(5)}{x-5}$ ✓

(see page 105)

$g'(5) = \displaystyle\lim_{x\to 5} \frac{g(x)-g(5)}{x-5}$

if $g(x) = \ln(x)$

2. Continuity and Differentiability (2.1)

(a) Let f be the function

$$f(x) = \begin{cases} x^2 - 1, & x \le 2 \\ 5x - 7, & x > 2 \end{cases}$$

$\lim_{x \to 2} x^2 - 1 = 3$

$\lim_{x \to 2} 5x - 7 = 3$

Which of the following statements about f are true?

I. f has a limit at $x = 2$ ✓ $\lim_{x \to 2} f(x) = 3$

II. f is continuous at $x = 2$ $f(2) = \lim_{x \to 2} f(x)$ ✓

III. f is differentiable at $x = 2$ ✗

$\lim_{x \to 2} 2x = 4 \neq \lim_{x \to 2} 5 = 5$

(b) Let f be the function

$$f(x) = \begin{cases} -1.5x^2, & x \le -2 \\ 6x + 6, & x > -2 \end{cases}$$

$$f'(x) = \begin{cases} -3x, & x \le -2 \\ 6, & x > -2 \end{cases}$$

Which of the following statements about f are true?

A. Continuous but not differentiable

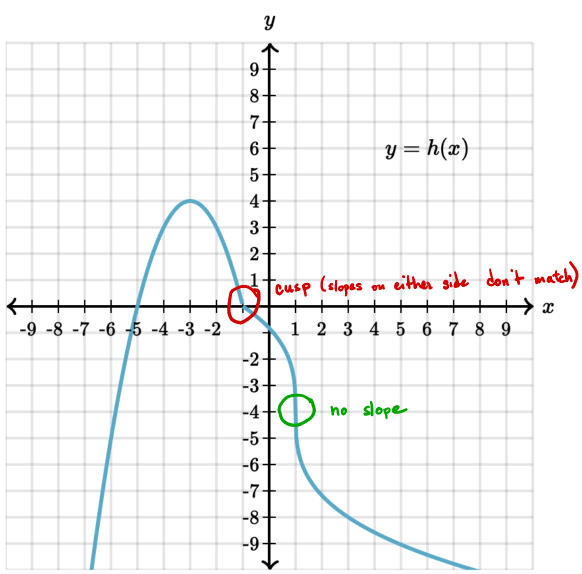
B. Differentiable but not continuous

C. Both continuous and differentiable ✓

D. Neither continuous nor differentiable

$\lim_{x \to 2} -3x = +6 = \lim_{x \to 2} 6$

Function h is graphed. The graph has a vertical tangent at $x = 1$.



Select all the x-values for which h is not differentiable.

$\lim_{x \to -1^-} h(x) \neq \lim_{x \to -1^+} h(x)$

so h is not differentiable at $x = -1$

vertical tangent line (no slope) at $x = 1$

3. Power / Product / Quotient / Chain Rule Practice (2.2, 2.3, 2.4)

(a) $\frac{d}{dx}\left(\frac{1}{\sqrt[4]{x^5}}\right) = \frac{d}{dx}\left(x^{-5/4}\right) = -\frac{5}{4}x^{-9/4}$

(b) $\frac{d}{dx}\left(\frac{1}{x^2} + \frac{1}{x} + x\right) = \frac{d}{dx}\left(x^{-2} + x^{-1} + x\right)$

$= -2x^{-3} - x^{-2} + 1$

(or $\frac{-2}{x^3} - \frac{1}{x^2} + 1$)

(c) $f(x) = (x^2 - 3x + 8)^3$

$f'(x) = 3(x^2 - 3x + 8)^2(2x - 3)$

chain rule

(d) $g(x) = (8x - 7)^{-5}$

$g'(x) = -5(8x - 7)^{-6}(8)$

(or $\frac{-40}{(8x-7)^6}$)

(e) $f(x) = \frac{x}{(x^2 - 1)^4} = x(x^2 - 1)^{-4}$ product rule & chain rule

$1(x^2 - 1)^{-4} + \left[-4(x^2 - 1)^{-5}(2x)\right]x$

(f) $F(v) = (17v - 5)^{1000}$

$F'(v) = 1000(17v - 5)^{999}(17)$

or $17000(17v - 5)^{999}$

(g) $k(r) = \sqrt[3]{8r^3 + 27} = (8r^3 + 27)^{1/3}$

$k'(r) = \frac{1}{3}(8r^3 + 27)^{-2/3}(24r^2)$

or $\frac{8r^3}{\sqrt[3]{(8r^3 + 27)^2}}$

(Q.R.)

(h) $H(x) = \dfrac{2x+3}{\sqrt{4x^2+9}}$

$$H'(x) = \frac{(2)(\sqrt{4x^2+9}) - \left[\frac{1}{2}(4x^2+9)^{-1/2}(8x)\right](2x+3)}{4x^2+9}$$

(or PR.)

$$H(x) = (2x+3)(4x^2+9)^{-1/2}$$

$$H'(x) = (2x+3)\left(-\tfrac{1}{2}(4x^2+9)^{-3/2}(8x)\right) + 2(4x^2+9)^{-1/2}$$

or $\dfrac{-12x+18}{(4x^2+9)^{3/2}}$

(i) $f(\theta) = \dfrac{\sin\theta}{\theta}$

$$f'(\theta) = \frac{(\cos\theta)(\theta) - (1)(\sin\theta)}{\theta^2}$$

(j) $g(t) = t^3 \sin t$

$$g'(t) = (3t^2)(\sin t) + (\cos t)(t^3)$$

(k) $h(z) = \dfrac{1-\cos z}{1+\cos z}$

$$h'(z) = \frac{(\sin z)(1+\cos z) - (-\sin z)(1-\cos z)}{(1+\cos z)^2}$$

or $\dfrac{2\sin z}{(1+\cos z)^2}$

(l) $f(x) = \dfrac{\tan x}{1+x^2}$

$$f'(x) = \frac{(\sec^2 x)(1+x^2) - (2x)(\tan x)}{(1+x^2)^2}$$

(m) $k(x) = \sin(x^2+2)$

$$k'(x) = \cos(x^2+2)(2x)$$

or $2x\cos(x^2+2)$

(n) $H(\theta) = \cos^5 3\theta = (\cos 3\theta)^5$

$$H'(\theta) = 5(\cos 3\theta)^4(-\sin 3\theta)(3)$$

or $-15\cos^4(3\theta)\sin(3\theta)$

(o) $g(z) = \sec(2z+1)^2$

$$g'(z) = \left[\sec(2z+1)^2 \tan(2z+1)^2\right]\left[2(2z+1)(2)\right]$$

or $(8z+4)\left(\sec(2z+1)^2\tan(2z+1)^2\right)$

(p) $f(x) = \cos(3x)^2 + \cos^2 3x = \cos(9x^2) + (\cos 3x)^2$

$$f'(x) = -18x\sin(9x^2) - 6(\cos 3x)(\sin 3x)$$

or $-18x\sin(9x^2) - 3\sin(6x)$

(q) $K(z) = z^2 \cot 5z$

$$K'(z) = 2z\cot(5z) - 5z^2\csc^2(5z)$$

(r) $h(\theta) = \tan^2\theta \sec^3\theta = (\tan\theta)^2(\sec\theta)^3$

$$h'(\theta) = 2(\tan\theta)(\sec^2\theta)(\sec^3\theta) + 3(\sec\theta)^2(\sec\theta\tan\theta)(\tan^2\theta)$$

or $2\tan\theta\sec^5\theta + 3\tan^3\sec^3\theta$

or $\sec^3\theta\tan\theta(2\sec^2\theta + 3\tan^2\theta)$

(s) $h(w) = \dfrac{\cos 4w}{1 - \sin 4w}$

$$h'(w) = \frac{-4\sin(4w)(1-\sin(4w)) + 4\cos(4x)(\cos 4w)}{(1-\sin 4w)^2}$$

(t) $f(x) = \tan^3 2x - \sec^3 2x$ $= (\tan 2x)^3 - (\sec 2x)^3$

$$f'(x) = 6\tan^2 2x \sec^2 2x - 6\sec^2 2x(\sec 2x \tan 2x)$$

or $6\tan(2x)\sec^2(2x)\left[\tan 2x - \sec 2x\right]$

(u) $f(x) = \sin\sqrt{x} + \sqrt{\sin x}$

$$(\cos\sqrt{x})\left(\tfrac{1}{2}x^{-1/2}\right) + \tfrac{1}{2}(\sin x)^{-1/2}(\cos x)$$

or $\dfrac{\cos\sqrt{x}}{2\sqrt{x}} + \dfrac{\cos x}{2\sqrt{\sin x}}$

(v) $g(x) = \dfrac{e^x}{x}$

$$g'(x) = \frac{xe^x - e^x}{x^2}$$

or $e^x\left(\frac{1}{x} - 1\right)$

or $\dfrac{e^x}{x}(1 - x)$

(w) $h(x) = x\ln x$

(x) This table gives select values of functions g and h, and their derivatives g' and h', for $x = -4$

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
-4	2	3	-1	5

Evaluate $\dfrac{d}{dx}(g(x) \cdot h(x))$ at $x = -4$.

$$= g'(x)h(x) + h'(x)g(x)\Big|_{x=-4}$$

$$= (-1)(3) + (5)(2)$$

$$= -3 + 10$$

$$= 7$$

(y) Let $k(x) = f(g(x))$.
If $f(2) = -4$, $g(2) = 2$, $f'(2) = 3$, $g'(2) = 5$, find $k(2)$, $k'(2)$, and the equation of the tangent line of k when $x = 2$.

point

$k(2) = f(g(2))$

$k(2) = f(2)$

$k(2) = -4$

slope

$k'(x) = f'(g(x)) \cdot g'(x)$

$k'(2) = f'(2) \cdot 5$

$k'(2) = 3 \cdot 5$

$k(2) = 15$

tangent line:

$$y + 4 = 15(x - 2)$$

(z) This table gives select values of functions g and h, and their derivatives g' and h', for $x = 3$

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
3	7	-2	8	4

Evaluate $\dfrac{d}{dx}\left(\dfrac{g(x)}{h(x)}\right)$ at $x = 3$.

$$= \frac{g'(x)h(x) - h'(x)g(x)}{[h(x)]^2}\Bigg|_{x=3}$$

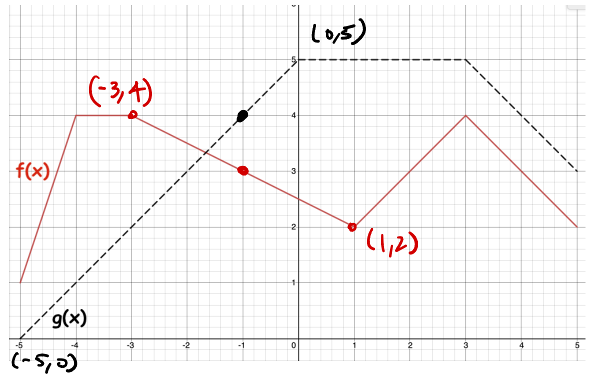
$$= \frac{8(-2) - 4(7)}{(-2)^2}$$

$$= \frac{-16 - 28}{4}$$

$$= -4 - 7 \qquad = -11$$

4. Differentiation Using Graphs
(AP style of question)

(a) Given the graph of f and g to the right, let $h(x) = f(x) \cdot g(x)$. Find $h'(-1)$



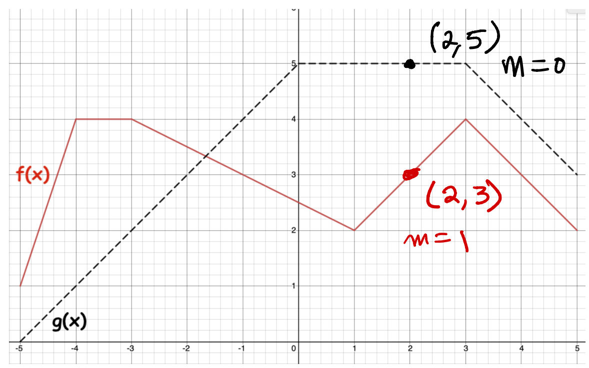
$$h'(x) = f'(x)g(x) + g'(x) \cdot f(x)$$

$$f(-1) = 3 \qquad g(-1) = 4$$

$$f'(-1) = \frac{-2}{4} = \frac{-1}{2} \qquad g'(-1) = 1$$

$$h'(-1) = \left(-\frac{1}{2}\right)(4) + (1)(3) = 1$$

(b) Given the graph of f and g to the right, let $k(x) = \dfrac{f(x)}{g(x)}$. Find $k'(2)$

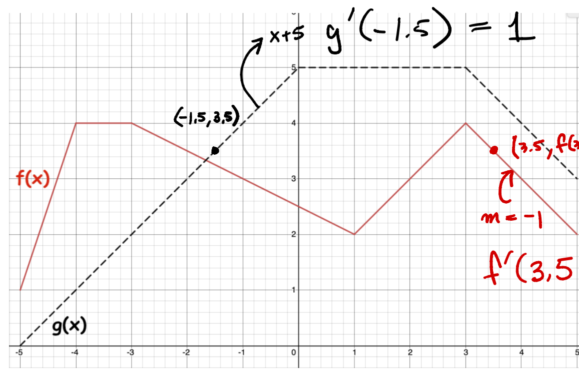


$$h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

	f	g
function	3	5
derivative	1	0

$$k'(2) = \frac{5 - 0}{25} = \frac{1}{5}$$

(c) Given the graph of f and g to the right, let $v(x) = f(g(x))$. Find $v'(-1.5)$

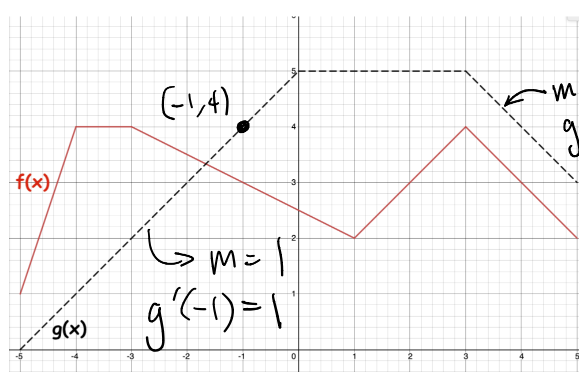


$$v'(x) = f'(g(x)) \cdot g'(x)$$

$$v'(-1.5) = f'(3.5)(1)$$

$$v'(-1.5) = (-1)(1) = -1$$

(d) Given the graph of f and g to the right, let $q(x) = g(g(x))$. Find $q'(-1)$



$$q'(x) = g'(g(x)) \cdot g'(x)$$

$$q'(-1) = g'(4) \cdot (1)$$

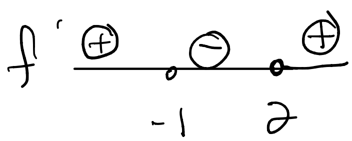
$$q'(-1) = (-1)(1) = -1$$

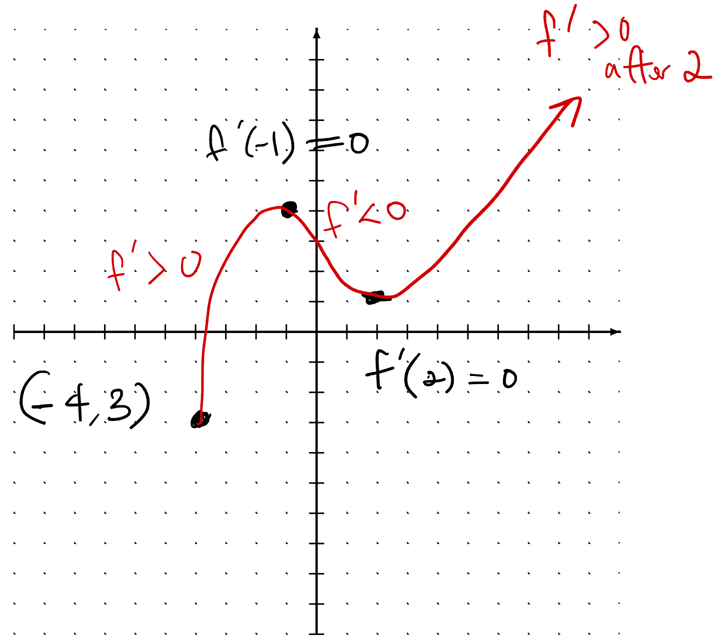
Many ways to do this correctly

5. Sketch the graph of each function, given the provided information.

(a) $f(-4) = 3$, $f'(-1) = 0$, $f'(2) = 0$,
$f'(x) > 0$ for $-4 < x < -1$,
$f'(x) < 0$ for $-1 < x < 2$,
$f'(x) > 0$ for $x > 2$,

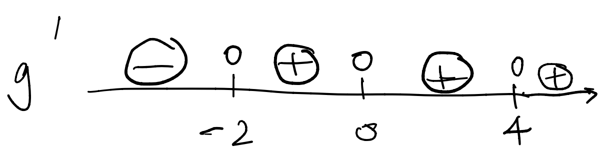
I like to organize this with a "sign line":

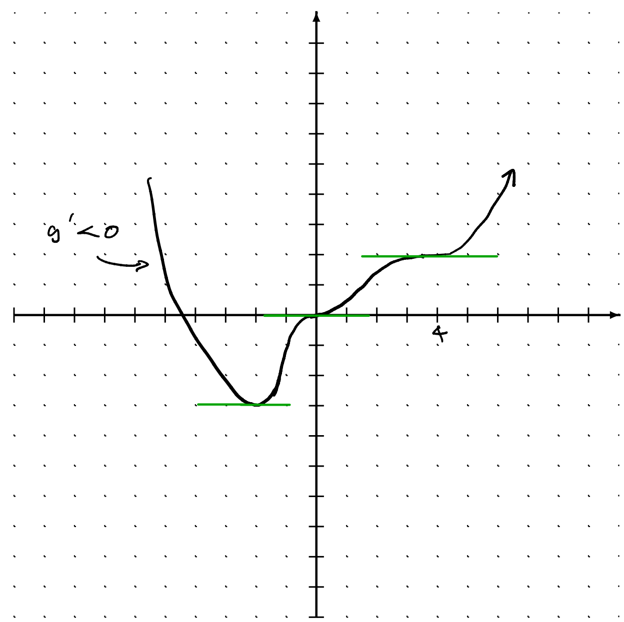




(b) $g(0) = 0$, $g'(0) = 0$, $g'(-2) = 0$, $g'(4) = 0$
$g'(x) > 0$ for $x \geq 0$,
$g'(x) < 0$ for $x < -2$,
$g'(x) > 0$ for $-2 < x < 0$,

Another "sign line":





6. Higher order derivatives (2.3)

(a) Find the second derivative of the function: $f(x) = (2x^4 + 8)^4$

$f'(x) = 4(2x^4+8)^3(8x^3)$

$f'(x) = 32x^3(2x^4+8)^3$

or $256x^3(x^4+4)^3$

$f''(x) = 96x^2(2x^4+8)^3 + 32x^3(3(2x^4+8)^2(8x^3))$

(b) Let s be the position (distance) function of a free falling object. Let s be defined to be

$$s(t) = -4.9t^2 + 120t + 45.$$

Find the velocity and acceleration of the object when it is 20 meters high. *(Assume the object was projected into the air at the time $t = 0$).*

$v(t) = -9.8t + 120$ m/s

$a(t) = -9.8$ m/s²

$v(20) = -9.8(20) + 120$ m/sec

or -76 m/sec

$a(20) = -9.8$ m/sec²

(c) Let $f(x) = x^8$.
Find $f''(x)$, $f^{(8)}(x)$, and $f^{(9)}(x)$

$f'(x) = 8x^7$

$f''(x) = 7 \cdot 8x^6$

$f'''(x) = 6 \cdot 7 8x^5$

$f^{(4)}(x) = 5 \cdot 6 \cdot 7 \cdot 8x^4$

$\vdots$

$f^{(8)}(x) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1\ x^{8-8}$

$f^{(8)}(x) = 8!$

$f^{(9)}(x) = 0$

(d) Let $y = \dfrac{1}{x}$. Find $\dfrac{d^3y}{dx^3}$

$y = x^{-1}$

$y' = -1x^{-2}$

$y'' = 2x^{-3}$

$y''' = -6x^{-4}$

or $\dfrac{-6}{x^4}$

(e) Let $y = 2e^{4x}$. Find $\dfrac{d^2y}{dx^2}$

A. $32e^{4x}$
B. $8e^x$
C. $40e^{6x}$
D. $\dfrac{e^{4x}}{8}$
E. $32x^2e^{4x}$

$y' = 8e^{4x}$

$y'' = 32e^{4x}$

(f) $h(x) = 6\ln(4x)$. Find $h''(x)$

$h'(x) = \dfrac{24}{4x} = \dfrac{6}{x} = 6x^{-1}$

$h''(x) = -6x^{-2}$ or $\dfrac{-6}{x^2}$

7. Implicit Differentiation (2.5)

(a) Find $\frac{dy}{dx}$ by implicit differentiation:

$$x^4 + 10x + 7xy - y^3 = 16$$

PR

$4x^3 + 10 + \left(7x\frac{dy}{dx} + 7y\right) - 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-4x^3 - 10 - 7y}{7x - 3y^2}$

(b) $2y^2 - x^2 + x^3y = 2$. Find $\frac{dy}{dx}$.

PR

A. $\frac{2x - 3x^2y}{4y + x^3}$

B. $\frac{2x}{4y + 3x^2}$

C. $\frac{2x - 4y}{3x^2}$

D. $\frac{4y + x^3}{2x - 3x^2y}$

$4y\frac{dy}{dx} - 2x + 3x^2y + x^3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(4y + x^3) = 2x - 3x^2y$

$\frac{dy}{dx} = \frac{2x - 3x^2y}{4y + x^3}$

(c) Let $y^4 + 5x = 11$. Find $\frac{d^2}{dx^2}$ at the point $(2, 1)$

$4y^3\frac{dy}{dx} + 5 = 0$

$\frac{dy}{dx} = -\frac{5}{4}\cdot y^{-3}\Big|_{y=1} = -\frac{5}{4}$

$\frac{d^2y}{dx^2} = -\frac{5}{4}(-3y^{-4})\left(\frac{dy}{dx}\right) = \left(\frac{-5}{4}\right)\left(\frac{-3}{y^4}\right)\left(\frac{-5}{4y^3}\right)$

$\frac{d^2y}{dx^2}\Big|_{y=1} = -\frac{5}{4}\left(\frac{-3}{1^4}\right)\left(-\frac{5}{4}\right) = \frac{-75}{16}$

(d) $3y^2 + x^2 - xy = \pi$. Find $\frac{dy}{dx}$.

PR

A. $\frac{y - 2x}{6y + x}$

B. $\frac{1 - 2x}{6y + 1}$

C. $\frac{y - 2x}{6y - x}$

D. $\frac{1 - 2x}{6y - 1}$

$6y\frac{dy}{dx} + 2x - \left(x\frac{dy}{dx} + y\right) = 0$

$\frac{dy}{dx} = \frac{-2x + y}{6y - x}$

$\frac{dy}{dx} = \frac{y - 2x}{6y - x}$

(e) $4x - x^2y + y^3 = 10$ Find the value of $\frac{dy}{dx}$ at the point $(1, 2)$

PR

$4 - \left(2xy + x^2\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx}(-x^2 + 3y^2) = -4 + 2xy$

$\frac{dy}{dx} = \frac{2xy - 4}{3y^2 - x^2}\Big|_{(1,2)} = \frac{0}{12 - 1} = 0$

(f) (challenge)

Let $xy = 18$. Find $\frac{dx}{dt}$ when $x = 2$ and $\frac{dy}{dt} = -6$.

PR

$x\frac{dy}{dt} + y\frac{dx}{dt} = 0$

$2(-6) + 9\frac{dx}{dt} = 0$

$\frac{dx}{dt} = \frac{12}{9} = \frac{4}{3}$

8. Tangent Lines (an application of Derivatives)

(a) The tangent line to the graph of function f at the point $(5,7)$ passes through the point $(1,-1)$. Find $f'(5)$

$f'(5) = \dfrac{7-(-1)}{5-1} = \dfrac{8}{4} = 2$

($f'(x)$ is the slope of the tangent line at x)

$y+1 = 2(x-1)$

(b) Let $y = \dfrac{1-2x}{3x^2}$ What is the equation of the tangent line at $(1, -\frac{1}{3})$?

$y = \frac{1}{3}x^{-2} - \frac{2}{3}x^{-1}$

$y' = -\frac{2}{3}x^{-3} + \frac{2}{3}x^{-2} \Big|_{x=1} = 0$

Tangent line is $y = -\frac{1}{3}$

or

$y' = \dfrac{3x^2(-2) - (1-2x)(6x)}{9x^4}$

$\dfrac{dy}{dx}\Big|_{x=1} = \dfrac{-6 - 6(1-2)}{9} = \dfrac{-6-(-6)}{9} = 0$

Tangent line: $y = 0x - \frac{1}{3}$

(c) Let $y = -x^3 + 4x^2$ What is the equation of the tangent line at the point where $x = 3$?

$y' = -3x^2 + 8x$

$y'\Big|_{x=3} = -3(9) + 8(3)$

$= -27 + 24$

slope $= -3$

$y(3) = -27 + 36 = 9$

$y - 9 = -3(x-3)$

(d) Let $y = \cot(x)$ What is the equation of the tangent line at $x = \frac{\pi}{6}$?

$y' = -\csc^2 x =$

$\dfrac{dy}{dx}\Big|_{x=\frac{\pi}{6}} = \dfrac{-1}{(\sin\frac{\pi}{6})^2} = \dfrac{-1}{(\frac{1}{2})^2} = -4$

Tangent line: $y - \sqrt{3} = -4(x - \frac{\pi}{6})$

(e) $x + \overset{PR}{2xy} - y^2 = 2$. Find the slope of the tangent line at the point $(2,4)$.

A. $\frac{3}{2}$

B. $\frac{9}{4}$

C. $\frac{1}{2}$

D. $-\frac{9}{4}$

$1 + \left(2y + 2x\dfrac{dy}{dx}\right) - 2y\dfrac{dy}{dx} = 0$

$\dfrac{dy}{dx} = \dfrac{-1-2y}{2x-2y}$

$\dfrac{dy}{dx}\Big|_{(2,4)} = \dfrac{-1-8}{4-8} = \dfrac{-9}{-4}$

(f) Use implicit differentiation to find an equation of the tangent line to the ellipse

$$\frac{x^2}{2} + \frac{y^2}{98} = 1$$

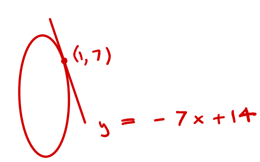
at the point$(1,7)$

$\dfrac{\cancel{2}x}{\cancel{2}} + \dfrac{2y}{\cancel{98}_{49}}\dfrac{dy}{dx} = 0$

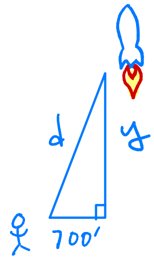
$\dfrac{dy}{dx} = \dfrac{-49x}{y}\Big|_{(1,7)} = \dfrac{-49}{7} = 7$

$y = -7(x-1) + 7$

graph:



9. An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?



d = distance between rocket & observer

y = height of rocket

① $\frac{dy}{dt} = 900$ ft/sec

② find $\left.\frac{dd}{dt}\right|_{y=2400}$, $d = \sqrt{700^2+2400^2} = 2500$

③ $700^2 + y^2 = d^2$

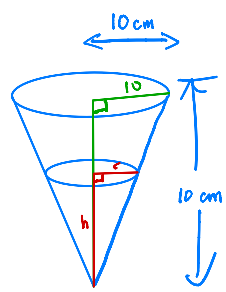
④ $0 + \cancel{2}y\frac{dy}{dt} = \cancel{2}d\frac{dd}{dt}$

⑤ $(2400)(900) = (2500)\left(\frac{dd}{dt}\right)$

$\left.\frac{dd}{dt}\right|_{d=2500} = \frac{(2400)(900)}{(2500)} = 864$ feet per second

The distance between the rocket and the observer is increasing at a rate of 864 feet per second when the rocket is 2400 feet from the ground

10. A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is the volume of water growing at the moment when the water level is 8 cm?. The volume of a cone is given by $V = \frac{\pi}{3}r^2 \cdot h$



h = water level

r = radius of water surface

By similarity

$\frac{r}{10} = \frac{h}{10}$

So $r = h$

(we can substitute this so we have an equation with one varible)

① $\frac{dh}{dt} = 2$ cm/sec

② find $\left.\frac{dV}{dt}\right|_{h=8}$,

③ $V = \frac{\pi}{3}r^2h$, here $r = h$ by similar Δ's

Since $r = h$ we have Volume based on height alone:

$V = \frac{\pi}{3}h^3$

④ $\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

⑤ $\left.\frac{dV}{dt}\right|_{h=8} = \pi 8^2(2) = 128\pi$ cm³/sec

When the water level is 8 cm high, the volume of the water is increasing at a rate of 128π cubic cm per second.